# Interpretation of automated forward modeling parameters for sawtooth events and substorms

M. Connors, R. L. McPherron, and C. R. Clauer

**Abstract**: Automated Forward Modeling (AFM) is an inversion technique based on magnetic data alone, which can indicate physical parameters associated with electrojets. From perturbations along a meridian, the total electric current crossing the meridian may be determined, as well as the latitudes between which it flowed. The technique is based on nonlinear optimization of the parameters of a forward model. It is possible to compare model output to the original input to ensure that the routine has functioned well and that output parameters are reliable and presumably have physical meaning. Characteristic behaviors of substorms are readily seen in modeling output: the current strengthens rapidly and considerably at an expansive phase onset, following a growth phase during which the electrojet borders move equatorward, usually with some strengthening of current. At onset the poleward border is often seen to move poleward rapidly. Poleward border activity may be noted then and also at other times. After an onset, the recovery phase is often marked by a retreat of the equatorward border, indicating the well-known shrinkage of the auroral oval then. These complete cycles of activity are absent in sawtooth events. These have many of the characteristics of expansive phase onsets, but maximum poleward expansion of the poleward border is followed by equatorward movement reminiscent of a growth phase. Since this is correlated with the interplanetary magnetic field remaining southward, the difference from common expansive phase phenomenology may simply be the lack of a recovery phase.

Key words: Sawtooth Events, Substorms, Inversion Techniques.

# 1. Introduction

The complete cycles of growth phase, expansive phase, and recovery typical of substorms are absent in sawtooth events. They show many of the characteristics of growth and expansive phases, but maximum poleward expansion of the poleward border is often immediately followed by equatorward movement reminiscent of a growth phase. We do not directly consider interplanetary magnetic field in this work, but it has been noted that sawtooth behavior is often correlated with a persistent IMF southward condition. The difference between sawtooth behavior and common expansive phase phenomenology may simply be the lack of a recovery phase due to continued forcing. Our work in modeling sawtooth events suggests many similarities to substorms. The most notable difference is that the currents across the active meridian (usually near midnight) are rather intense compared to those of typical substorms.

# 2. Automated Forward Modeling (AFM)

Interpretation of ground magnetic data is difficult, even if the data come from the same magnetic meridian. Examples of magnetic data from many locations are common in the literature, or one may examine the solid lines in Fig. 5 or Fig. 7. In those figures the X (northward) and Z (downward) com-

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**Fig. 1.** By varying the current (large horizontal arrow) and latitudinal boundaries (small vertical arrows) an optimal match of model results and data can be made and these physical parameters determined. Field-aligned currents may be included (as shown at ends of ionospheric current flow region).

ponents of the magnetic field are shown from the six stations on the Canadian Churchill meridian most relevant to studying auroral zone currents. Automated Forward Modeling proposes a forward model of current systems which could give rise to the magnetic perturbations observed. The parameters in that model are varied in such a way that the deviation between the observed magnetic fields and those predicted by the model are reduced. In the ideal case, the parameters can be chosen to correspond to simple physical parameters associated with the current system. A forward model can be made using the Biot-Savart law in combination with Earth induction, by specifying where currents flow in space and the ionosphere [6] [7]. Adjustment of the parameters specifying the current system can be done until the match to the input data is optimal. In principle, arbitrarily complex current systems may described in three dimensions in near-Earth space and their parameters determined. In practice, available magnetic data is sparse and well-determined solutions can be difficult to obtain.

The optimum situation can be found when data from me-

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**Fig. 2.** Growth phase and onset of a substorm on June 3 1997. Upper panel shows the electrojet north and south boundaries as it traversed the Churchill meridian. Bottom panel shows total current across the meridian. The growth phase is clear. Subsequent substorm onset is somewhat unusual in being well poleward of the region of growth phase currents. However in general this event is illustrative of the strength of a typical substorm.

ridian chains is available, since in many cases a simple model involving an electrojet flowing across the meridian chain is physically realistic, and in this case there is a good ratio of data available to parameters to be determined. From meridian chain data a forward modeling procedure can give the current across the meridian and the latitudes between which it flowed. In this way the many data points specifying the magnetic perturbations along a meridian can be reduced to three simple parameters, which have an easily understood physical significance. For efficient processing of large amounts of magnetic data, the matching process can be automated. In the Automated Forward Modeling (AFM) procedure this is done using the Levenberg-Marquardt algorithm [9]. A schematic of the variables involved and the way in which they are varied is shown in Fig. 1. Detailed descriptions of the AFM procedure are given elsewhere [4] [5].

# 3. Substorms

AFM has been applied on the CANOPUS Churchill meridian  $(336^{\circ} \text{ mag.}; \text{ station latitudes shown on graph) to invert$ the whole year of 1997. We do not show the input data, butoutput initially chosen as to show growth, expansion, and somededegree of recovery is shown in Fig. 2. This event from June3 1997 does show some atypical features deserving followup, but a strong growth phase is visible from 5.3 UT to 6.0 UT (between vertical bars). The onset at 6.0 is mainly marked by a poleward leap of the current, which subsequently strengthens over about one half hour to 1 MA. This is at the upper end of what is typical of a substorm: currents usually are below 1 MA, and the latitudinal range expands rapidly to over  $10^{\circ}$ , with rapid poleward motion of the poleward border just after onset. The overal time scale for substorm-associated enhanced currents is of order one hour.

Extended Churchill Line October 29 2003



**Fig. 3.** Superstorm activity on October 29 2003. Upper panel shows the electrojet north and south boundaries as it traversed the extended Churchill meridian. Bottom panel shows total current across the meridian.

### 4. Superstorm Onset

The "Hallowe' en storm" of October 2003 featured currents on Oct 29, 2003, which can be regarded as being at the upper limit of those associated with substorm activity. The electrojets extended to rather low latitude, so that data had to be used from an extended Churchill line featuring Cambridge MN (X only), Boulder, and Tucson. Accurate modeling down to  $40^{\circ}$  magnetic latitude was done as shown in Fig. 3. A recognizable growth phase took place starting at about 5 UT. This is seen through the steady equatorward motion of the electrojet boundaries (upper panel) by about 3 degrees until about 6:15 UT. At that time, following a possible large impulsive current, the current rose steadily and the electrojet poleward border moved rapidly poleward. By 7 UT, the electrojet was about  $15^{\circ}$  wide with nearly 6 MA of current. Other aspects of the onset are like those of substorms, so this may be regarded as a very large substorm onset. This large current may be used for comparison with that during sawtooth events.



**Fig. 4.** Sawtooth event of October 4 2000. Upper panel shows the electrojet north and south boundaries as it traversed the Churchill meridian. Bottom panel shows total current across the meridian.Vertical lines are onset times deduced from Pi 2 pulsations.

#### 5. Sawtooth Events

The typical signature of sawtooth events as seen in ground magnetic signatures is recurrent X bays initiated rapidly, like substorms.

#### 5.1. October 4, 2000 Sawtooth Event

Fig. 5 shows sawtooth bays as the lower trace in each panel for the sawtooth event of October 4, 2000. The bays are present in various sizes at the different stations in this data from the Churchill meridian. Although the sawtooth nature is evident, examination of the data does not make it very clear what actually took place in terms of physical parameters. From satellite observations of energetic particle injections or ground observations of Pi 2 pulsations, onset times were determined for this event. Those times are indicated by vertical lines. Fig. 4 shows the results of AFM inversion of the data presented in Fig. 5. At times between 3 and 16 UT the scatter in the inversion results is minimal and they can be considered reliable. The onset times determined by other methods are again indicated; at these times the current strengthens rapidly and the poleward border moves rapidly poleward, features typical of substorm onset. Preceding each such onset during the period of reliable



**Fig. 5.** Sawtooth event of October 4 2000. Comparison of observed (solid) and model, with X black and Z blue (X generally the lower trace, Z generally upper). Vertical lines are onset times deduced from Pi 2 pulsations.

inversion, there is a growth phase with steady equatorward motion of the electrojet borders. Rise times for current are similar to those for substorms, and the repetition period is usually similar to that for the rise and decline of substorm total current. However, the total current across a meridian is generally larger for sawtooth events than for substorms, in this case up to 2.5 MA. It may be noted that even during the growth phase analogs, currents were at levels of approximately 0.5 MA, that is, stronger than in many typical substorm expansive phases.

Further examination of Fig. 5 shows the degree to which the AFM modeling has succeeded in representing the data from the six magnetic stations by three simple parameters. The X (generally lower) component data is shown by a solid line, while the X resulting from the model is shown by discrete points. At times from 0 UT to 18 UT the two agree very well. Some care has to be taken in interpreting this agreement when the perturbations are near zero. At such times the geometric parameters may not be well determined simply since there is basically no current upon which to base an inversion. Generally, as indicated above, when this happens there will be large scatter in the electrojet border parameters. However, between 3 and 16 UT both the match to X data and the lack of scatter suggest an excellent model fit. The Z component is also plotted (generally the upper trace). Here the fit is generally very good but not quite as excellent as that for X. This is attributed to the more rapid variation in Z when a station is near a current source in the ionosphere. Z can reflect structure in the electrojet which is not present in the simple model and thus is harder to match than is X.

#### 5.2. February 18, 1999 Sawtooth Event

Much as with the previous pair of figures, Fig. 6 and Fig. 7 present modeling results and comparison to data, in this case for the sawtooth event of February 18, 1999. This event was recently discussed in detail using magnetic and other sources of data [3] [8]. Once more it is generally clear when the results were valid, and the match to data was good for a large part of the UT day and of the event. The substorm-like pattern of expansive phase current intensification and poleward motion of the poleward border is clear, and in most cases a clear growth phase is seen. Expansive phase analog currents of up to about 3.5 MA were present, and the growth phase portions had currents averaging 1 MA, stronger than those of most substorm expansive phases. Once more the comparison of data and model output is very good, suggesting that the physical parameters derived are realistic.



**Fig. 6.** Sawtooth event of February 18, 1999. Upper panel shows the electrojet north and south boundaries as it traversed the Churchill meridian. Bottom panel shows total current across the meridian.Vertical lines are onset times deduced from Pi 2 pulsations.

# **5.3.** November 8, 2004: Sawtooth Event with Optical Data

Ground optical data was not available for the events described above. For an event on November 8, 2004, however, two meridian scanning photometers in the Churchill meridian were operative under clear skies. These instruments are located at the southern end of the chain at Pinawa and near the middle of the auroral zone at Gillam. The instrument further north at Rankin Inlet was not returning data on this date. The relative locations of these stations may be seen in the right hand part



**Fig. 7.** Sawtooth event of February 18, 1999. Comparison of observed (solid) and model, with X black and Z blue (X generally the lower trace, Z generally upper). Vertical lines are onset times deduced from Pi 2 pulsations.

of Fig. 2, and their dipole magnetic coordinates read from the latitude scale. The meridian scan data from Pinawa and Gillam may be stacked timewise (each scan lasts 2 minutes) and placed one station above the other to cover approximately 1000 km along the meridian. Such keogram data is shown in Fig. 8 for comparison with magnetic inversion results. The latter must be regarded as preliminary since only Canadian data was used. Since the electrojets clearly extended rather far south, well beyond the  $60^{\circ}$  dipole magnetic latitude of Pinawa, there was no good constraint at the southern border. In this sense the predominance of low latitude activity suggested by Fig. 8 is deceptive: so much so that one of the optical plots has been used to cover some of the inversion results for latitudinal borders since they are not highly significant. Nevertheless, the total currents shown correlate very well with the optical intensifications. Between 5 and 6 UT, the electrojet had extended very far north and this is borne out by the optical data. At this time the indication of 6 MA across the meridian is likely quite accurate. At other times, the overestimation of the electrojet width leads to an overestimation of the current. However, the maximal current in this event approaches very closely that of the Hallowe'en superstorm. In both cases part of the reason for the large total current was the width of the electojet, allowing current to be carried over approximately a 20° band of latitude. This is supported in this sawtooth example by the optical data showing bright aurora extending past the horizons of both meridian scanning photometers.

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# 6. Conclusions

Very wide electrojets and large currents are a consistent feature of the sawtooth events studied, as indicated by modeling giving good agreement with station data across the Churchill meridian, and in the last case supported by optical data. Independent studies of related parameters have recently shown that the degree of dipolarization observed at geosynchronous orbit is larger for sawtooth events than for substorms in a statistically significant way [1]. Further, cross polar cap potential is larger [2]. These results are also consistent with our finding of very large currents across the modeling meridian. Our studies use local magnetic perturbations in the auroral zone (extended equatorward as needed and possible) and largely correspond to the effects of Hall currents in the local ionosphere. Studies including low-latitude perturbations conclude that the three-dimensional current system in a sawtooth event is likely similar to the of the three-dimensional substorm current wedge (SCW) usually associated with substorm onsets [8] but with a larger longitudinal extent than is typical of such onsets [8] [3]. We note that AFM can be used in to model SCW systems in a natural way, and could in principle answer some of the questions about the low-latitude perturbations, such as unusual D/H perturbation ratios, which arose from these recent studies.

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**Fig. 8.** Sawtooth event of November 8, 2004. Upper middle panel shows the electrojet north and south boundaries as it traversed the Churchill meridian. Bottom panel shows total current across the meridian. Meridian scanning photometer from Pinawa and Gillam superposed, 557.7 nm green line in lower middle, H beta emission at top. The color scale maximizes at red, which is 15 kR for 557.7, 70 R for H beta.

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